The space of ultrametric phylogenetic trees

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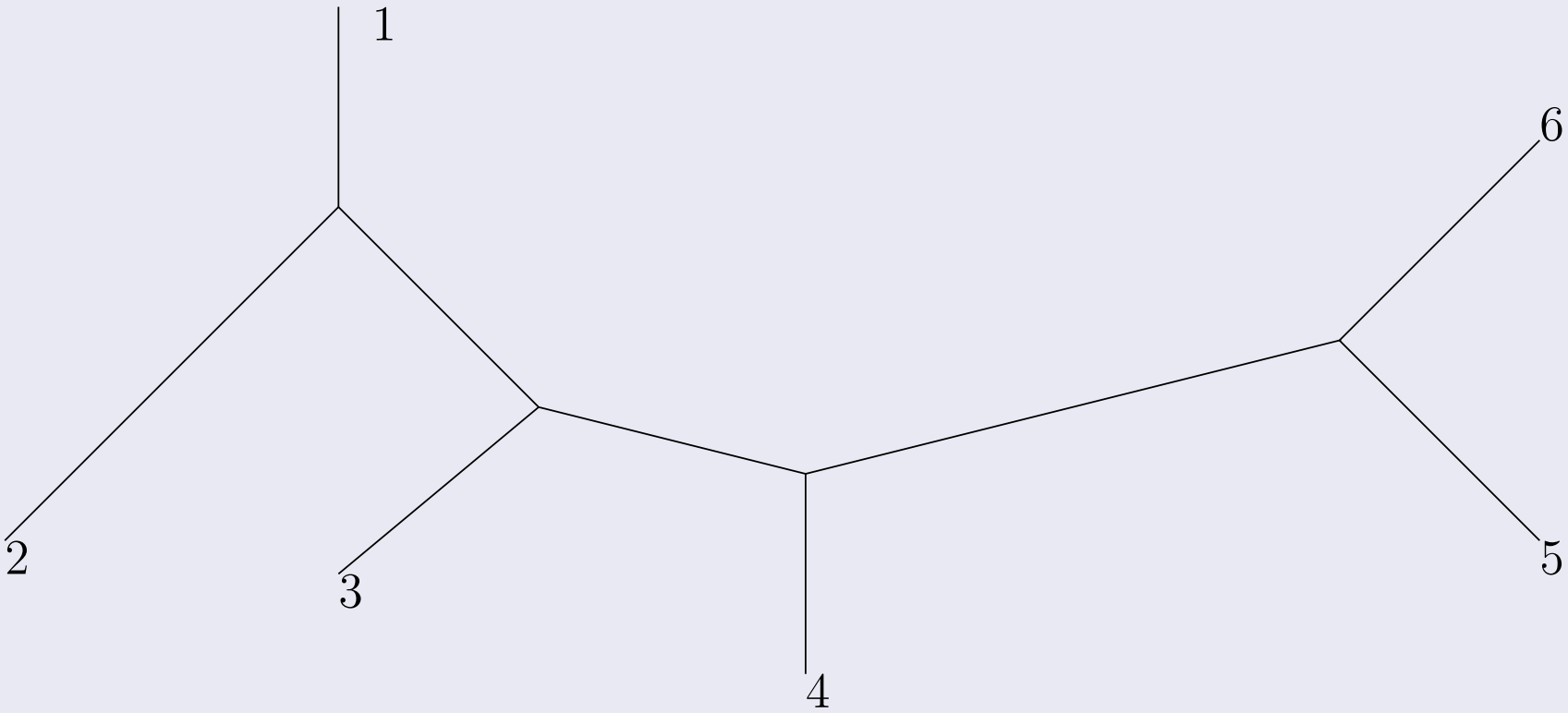
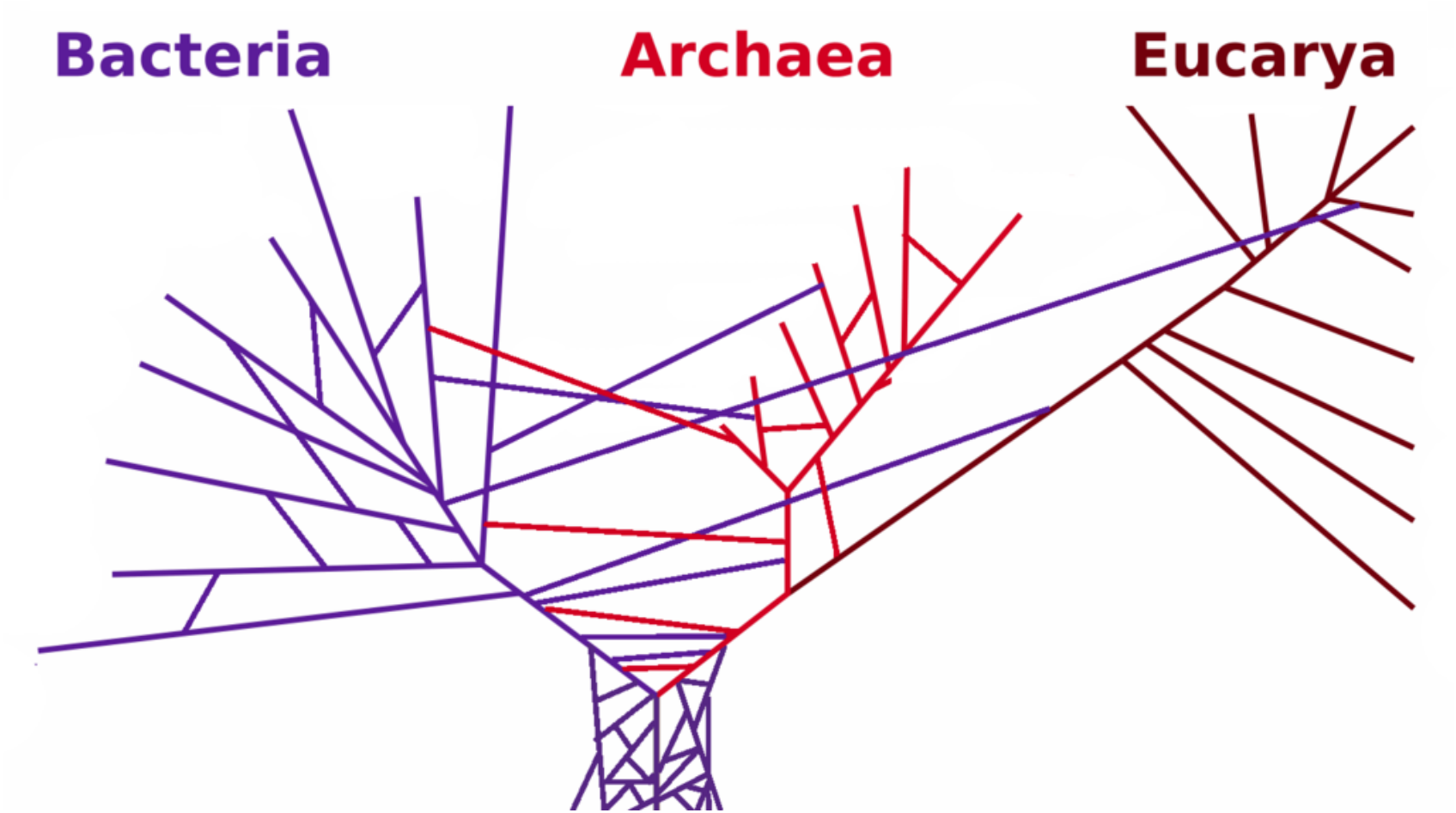
Unrooted phylogenetic tree

Definition



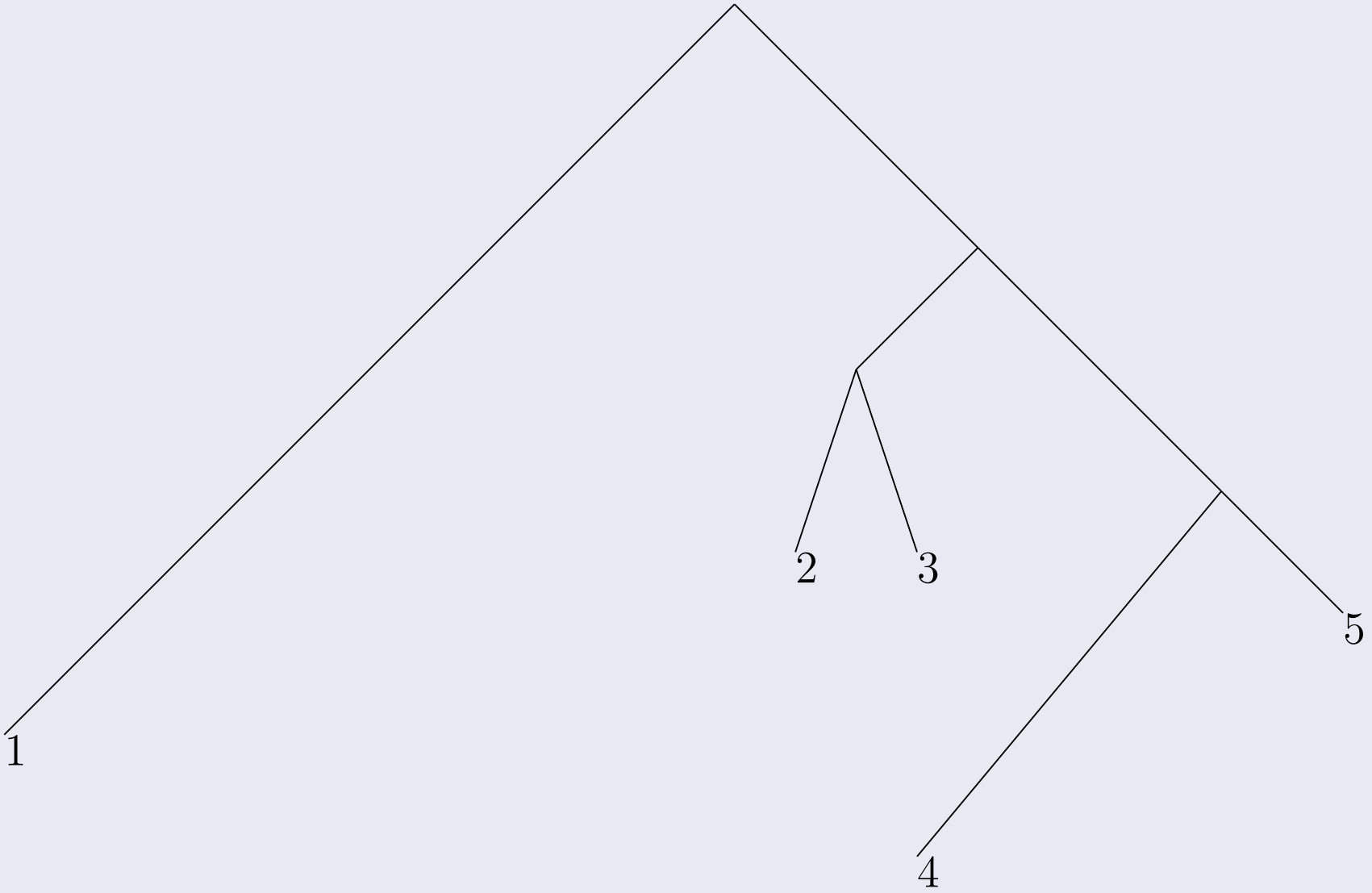
Real Life



Credit link: http://commons.wikimedia.org/wiki/File:PhylogeneticTree_horizontal_transfers.png?uselang=en-gb

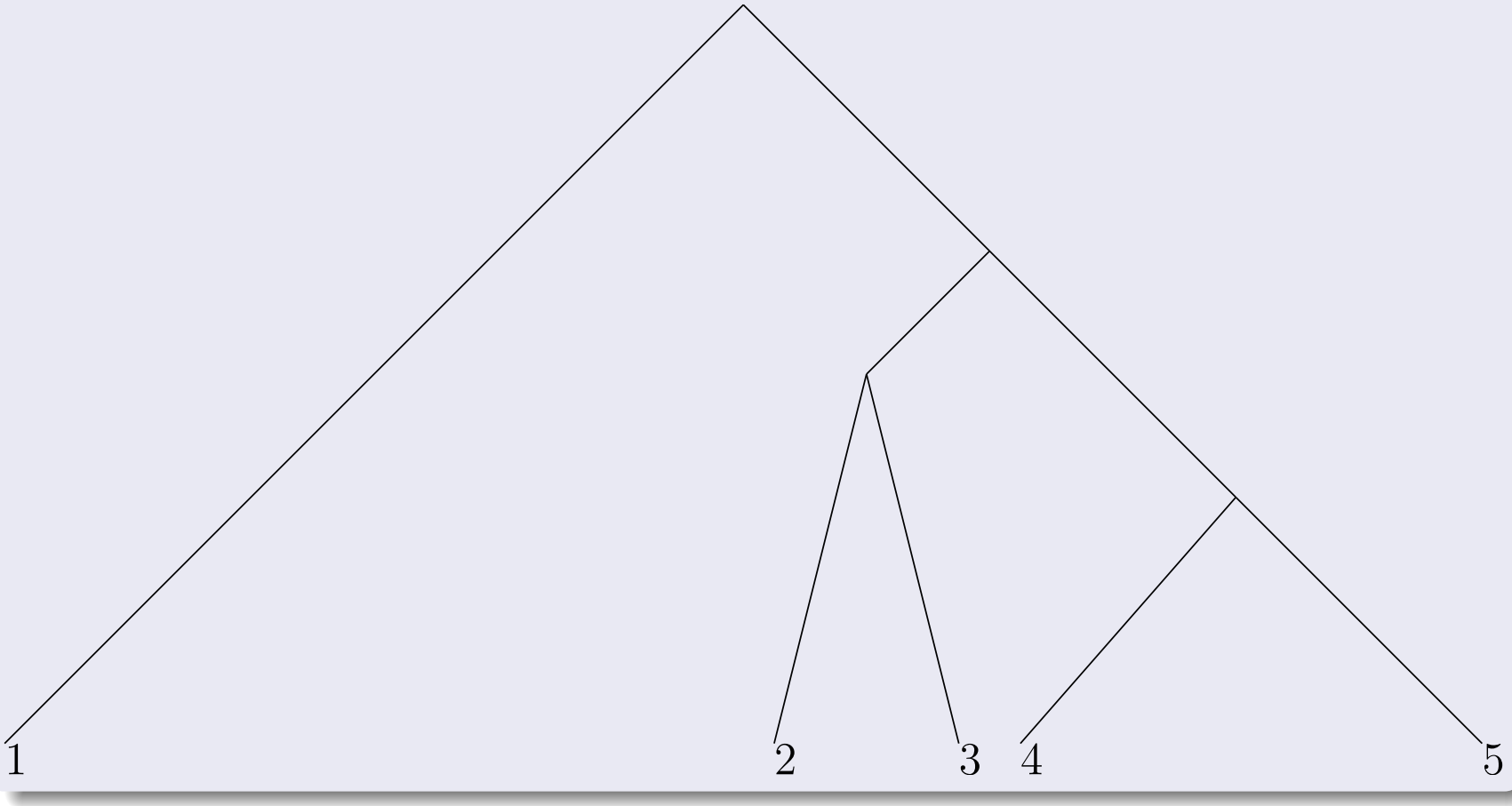
Rooted phylogenetic tree

Definition



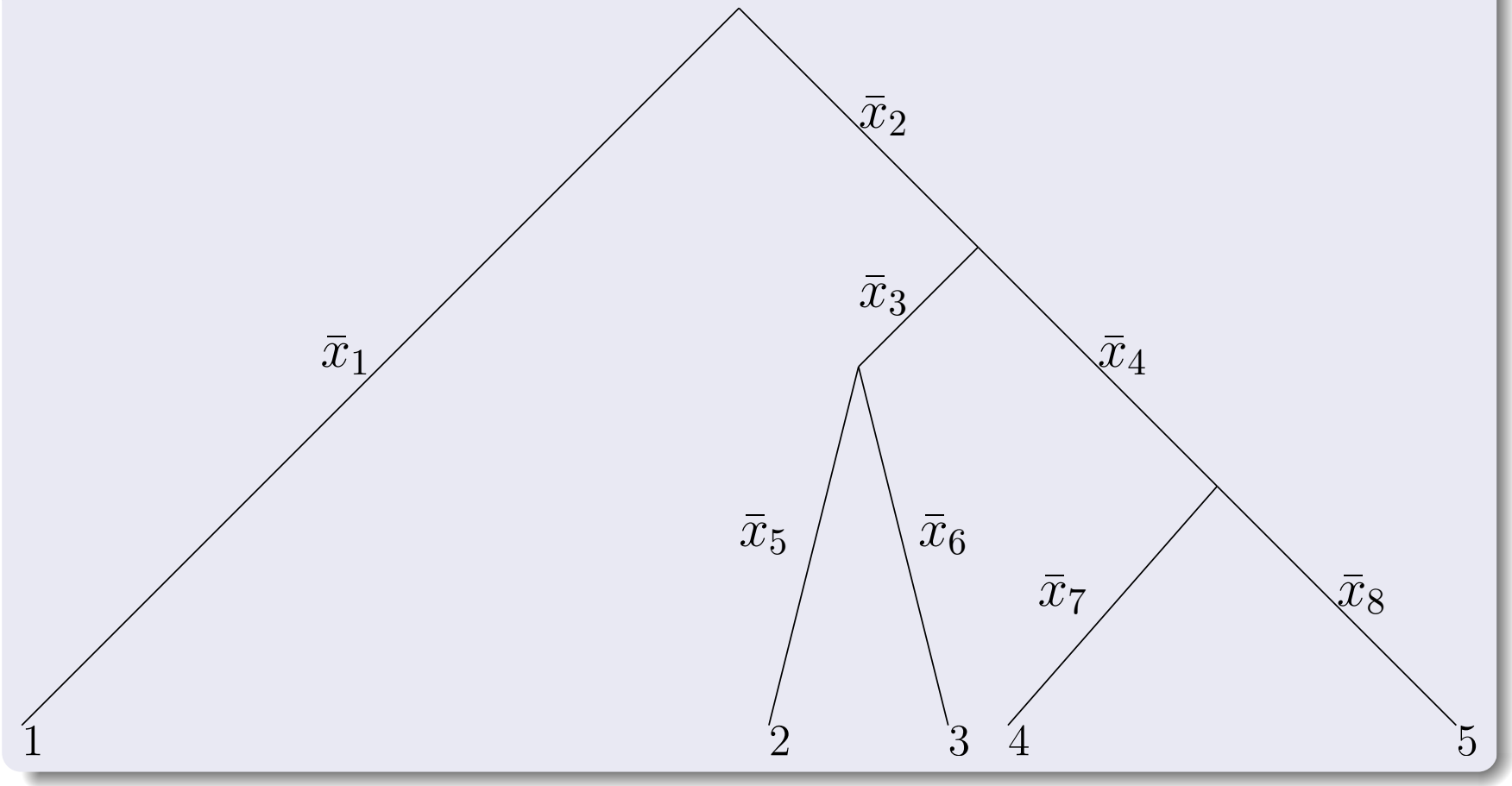
Equidistant (ultrametric) phylogenetic tree

Definition



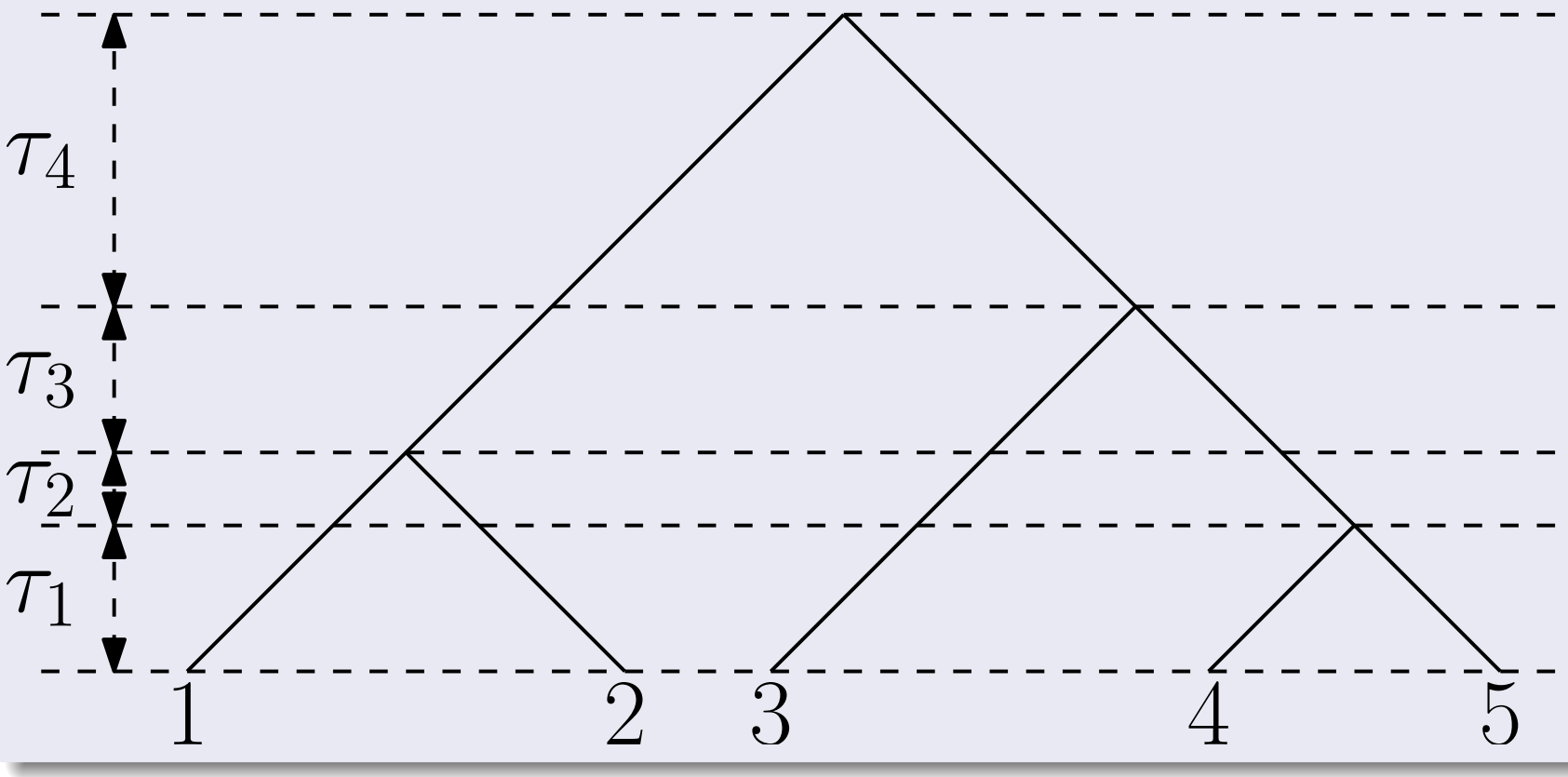
Equidistant phylogenetic tree with parameters

Definition



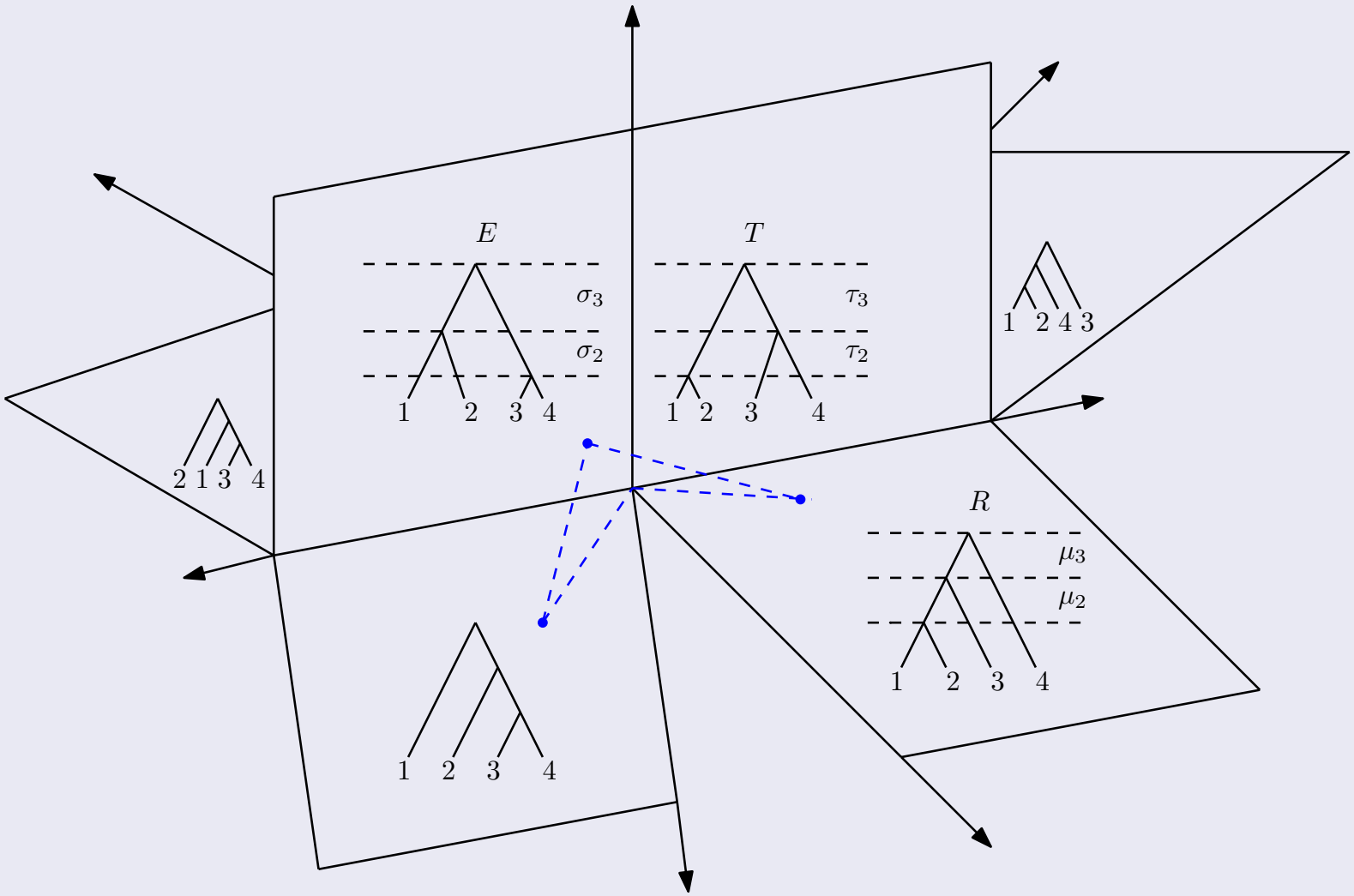
τ-parameterisation

Definition



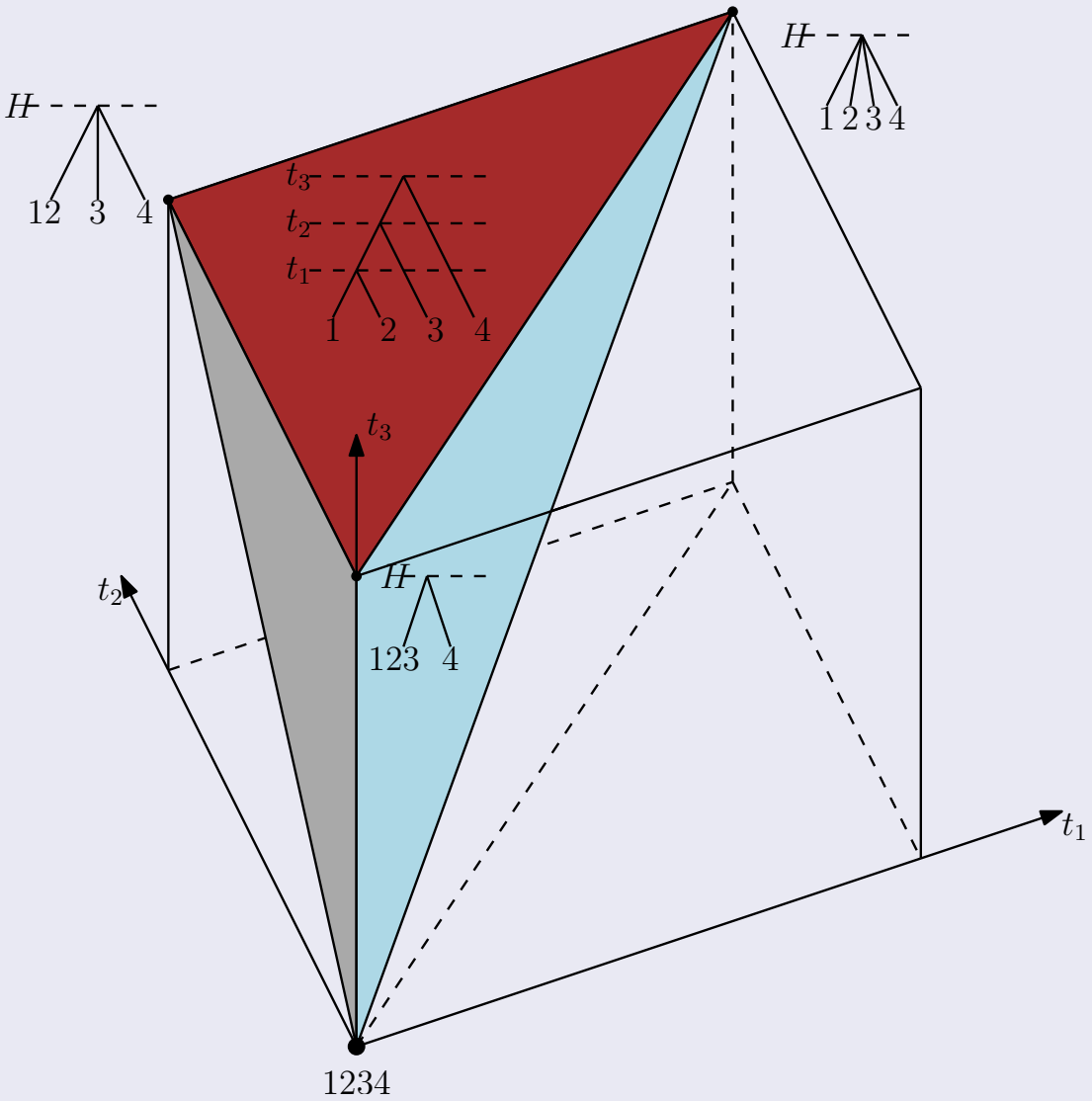
τ-space

Definition

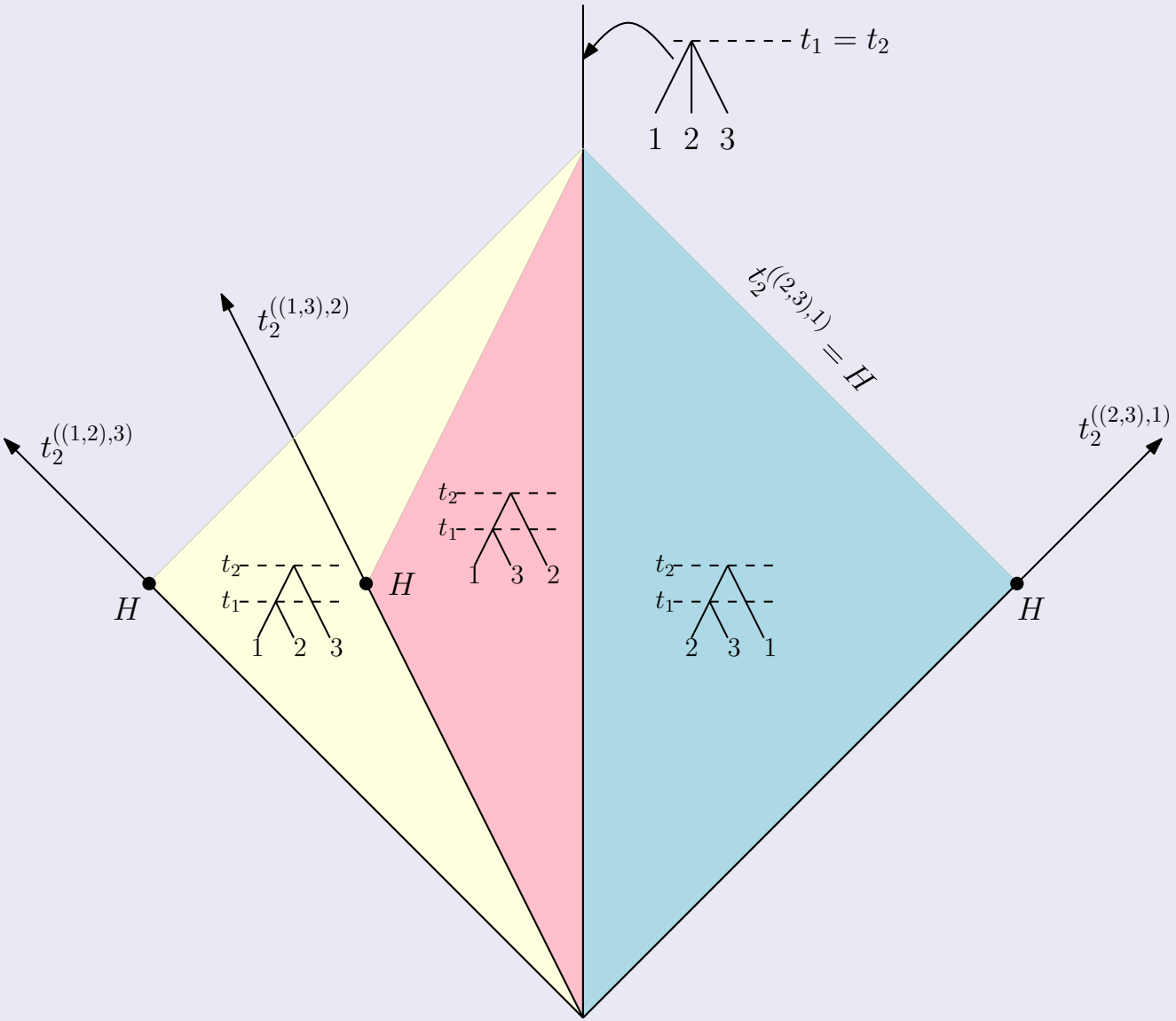


t-parameterisation

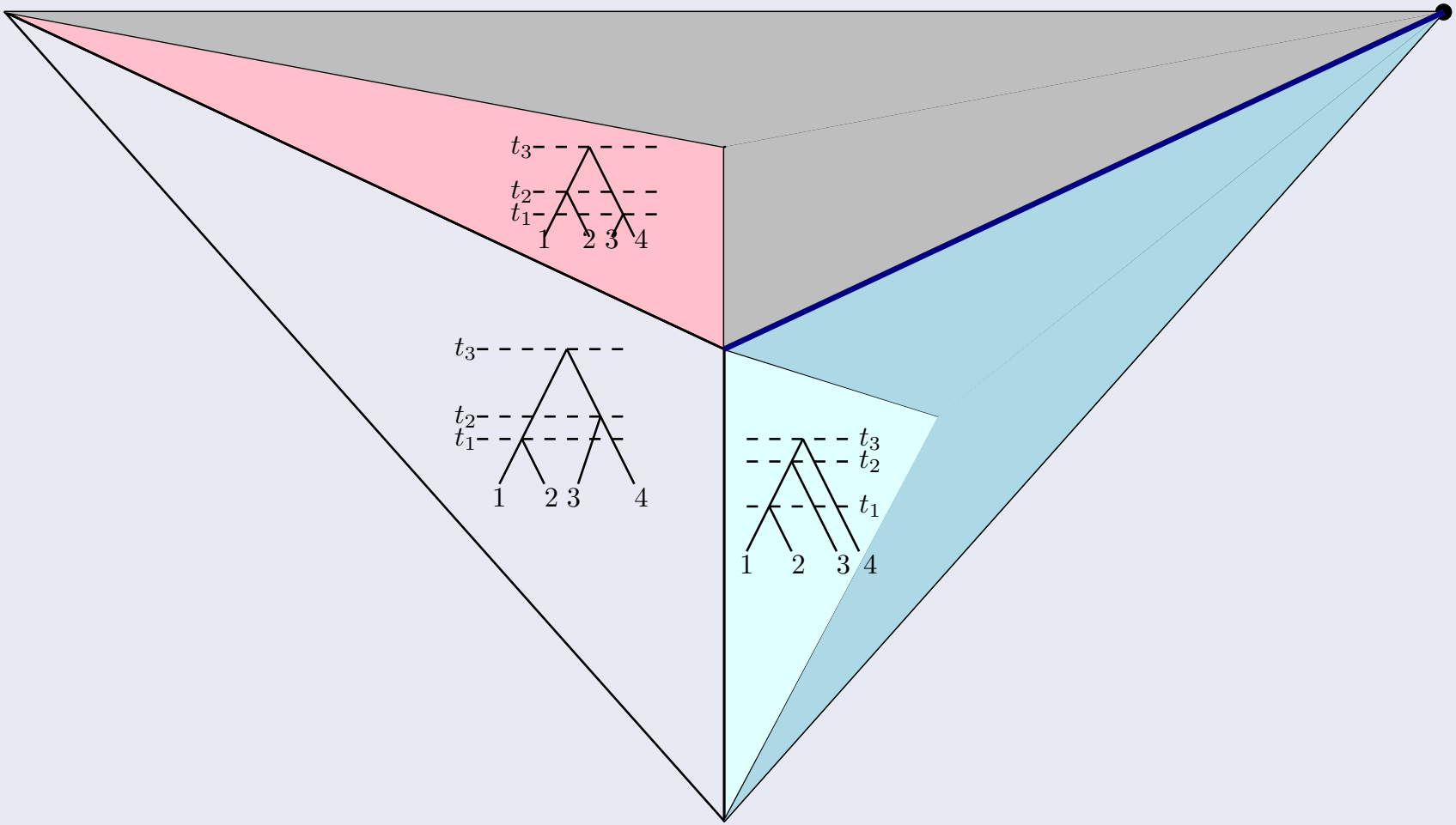
Definition



Definition


$t_1 = t_2$
1 2 3
$t_2^{((1,3),2)}$
$t_2^{((2,3),1)} = H$
$t_2^{((1,2),3)}$
$t_2^{((2,3),1)}$
t_2
t_1
1 2 3
H
t_2
t_1
1 3 2
H
t_2
t_1
2 3 1
H

Definition



Motivation

1. Bayesian MCMC: Mixing rate, access time, efficient proposals.
2. Summarising posterior: No need to introduce several random variables on different probability spaces, no need to fit inconsistent data together.
3. Interesting algorithmic/data structures problems: How to solve NP-complete problems on real computers for real data (Chris and Erick can compute SPR-distance).
4. Interesting geometries: "Every new example of a non-trivial simplicial complex of non-positive curvature is a big deal."

Geodesic is a short for shortest path.

Theorem (G and Drummond [6])

τ-space has unique geodesics.

The reason this is true is pretty much the same as why this is true in BHVspace [2].

Theorem (G and Drummond [6])

Geodesics in τ-space are efficiently computable.

(Assuming $\mathcal{O}(n^4)$ is efficient.)
The reason this is true is pretty much the same as why this is true in BHVspace [5].

Nice metric spaces

Definition

A metric space is called *nice* if most statisticians would like it.

Examples of nice metric spaces include real line, Euclidean space, and its nice subspaces.
Examples of not nice metric spaces include all non-measurable subsets of a Euclidean space, all nowhere dense subsets of a Euclidean space, and most importantly the spaces where it is hard to define a random variable.

Theorem (Billera, Holmes, and Vogtmann [2])

The space of phylogenetic trees is a nice space.

Theorem (G and Drummond [6])

The space of equidistant phylogenetic trees is a nice space.

Parameterisation matters!

Theorem (G and Drummond [6])

t*-space is not a very nice space.*

That is,

Theorem (G and Drummond [6])

Geodesics in t*-space are hard to compute. Possible but hard.*

Hard here means that we (Alexei and I) don't know how.

More formal

Definition

A geodesic metric space is called *nice* if it is a convex path-connected subspace of a computable metric space with unique geodesics of the same dimension.

Theorem (G and Drummond [6])

τ-space is an efficiently computable cubical complex with unique geodesics.

Conjecture (G and Drummond [6])

t-space is a simplicial complex with unique geodesics, which are NP-hard to compute.

Corollary

Both τ-space and t-space are nice.

Thank you for your attention!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